Posing Cognitively Demanding Tasks to All Students

Rachel Lambert and Despina A. Stylianou

KRISTIAN SEKULIC/ISTOCKPHOTO

One middle school teacher developed classroom routines to make challenging questions accessible to all learners in her class.

Cognitively demanding tasks (CDT) (Stein et al. 2000) are necessary for the development of students' mathematical reasoning skills. Research is unequivocal on the importance of giving students opportunities to engage in such tasks. Starting with the work of the QUASAR project, it has been shown that classrooms in which students engage in challenging tasks exhibit the highest learning gains (e.g., Silver and Stein 1996).

Although current reform efforts call for mathematics learning for *all* students, learners who struggle in mathematics or who have special education placements have less access to demanding mathematics (Weiss et al. 2003).

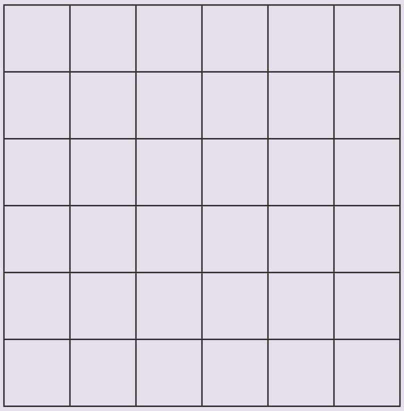
To successfully include all learners in the mathematics classroom, instruction needs to be designed so that it is accessible to all without compromising cognitive demand. How can we design instruction so that students of varying levels and abilities can develop strategies that work for them, understand the strategies of others, and make thoughtful choices about which strategies to use? We describe one episode that occurred in the classroom of Ms. Ramos, in which instruction was designed for an inclusion classroom to embrace learners with disabilities. Using the Universal Design for Learning (UDL) (CAST 2011), one approach to designing widely accessible instruction from the outset, we present an analysis of how this teacher created an accessible classroom focused on answering challenging questions.

WITNESSING AN INCLUSION CLASSROOM

Early in October of their seventh-grade year, students were working on rational numbers, tackling percentages for the first time in this course. The

Fig. 1 This grid problem, posed to students at the beginning of class, elicited discussion in Ms. Ramos's class.

Shade 25 percent of the grid below.



1. How many squares did you shade?

2. What fractional part of the area was shaded?

Source: Adapted from Stein et al. (2009)

problem projected on the whiteboard asked them to shade part of a grid (see **fig. 1**). Ramos read the problem and asked students to work on the printed pages distributed to the class. Students worked independently on the problem, and then discussed their strategies with partners. After a few minutes, she asked the students to share their answer, reminding them that the class needed to know "how you arrived at your answer." Several solutions were given:

- Erika figured out that the grid consisted of 36 squares. She found 50 percent, or 18, and halved it again to find 9 squares. She then made a horizontal drawing.
- Solomon broke the 6×6 grid into four pieces, first halving and then halving again. He, too, found that 25 percent was 9 squares. He colored a 3×3 grid.
- Carlos, who multiplied 36×25, "put the decimal in its place," using the trick of moving the decimal two places to the left because the 25 would really be 0.25. Next Manuel shared that he found 10 percent of 36, then another 10 percent, then 5 percent, and finally added these percentages to find 25 percent.

While each student was speaking, Ramos represented the strategy on the board, waiting patiently when students were thinking or struggling to articulate their thinking. She then verbally summarized each strategy. When Manuel shared, Ramos made a connection from his logic to Erika's strategy of finding an easy percentage. Ivana raised her hand and questioned the effectiveness of Erika's strategy, "I think that Erika's rule is not always gonna work because what if you get a number that is not so easy to work with? It would be kinda difficult if it didn't split." A discussion ensued about whether *any* number can be split in half. Monica volunteered that she used division, a procedure that she had learned in a previous classroom, and wanted to know why it did not work.

As multiple students raised their hands to address Monica's question, we got a glimpse of a classroom community in which students shared their own strategies, worked to understand the strategies of others, and made choices about their efficiency and effectiveness. In this classroom, mathematical practices advocated by the Common Core State Standards for Mathematics came to life: Students made sense of problems and persevered in solving them, constructed viable arguments, listened and critiqued one another, and made sense of quantities and their relationships. In other words, students were genuinely engaged in mathematical activity.

EMBRACING DIVERSE LEARNERS

This was an inclusion classroom in a large city, equally split between students with and without special education labels. Ninety-two percent of students at the school were Latino. Of the six students mentioned, four had Individual Education Plans that included goals in mathematics. Four of the students who participated were labeled with learning disabilities. All but one student mentioned were bilingual in Spanish and English, although none were currently designated as English language learners.

One way to address all these differences is to target each student's differences and differentiate for those difficulties, presenting each learner with a task specifically designed for that learner alone. We argue, however, for a different approach to teaching a wide range of learners: To successfully include all learners in the mathematics classroom, instruction might be designed in such a way that it is accessible for the widest net of learners from the beginning. Toward this end, we use the principles of UDL (CAST 2011).

UDL was designed on the basis of emerging neuroscience: Learning is not based in localized areas or consistent across individuals (CAST 2011; Meyer and Rose 2005). All learners show variation across subjects and contexts. Learning a complex topic like mathematics is distributed across the brain, with individual students processing information in various ways. With so much diversity in neuroprocessing, even students with disabilities are as dissimilar as they are similar; the only response to such diversity is to plan accessible curriculum from the beginning rather than responding to each difference.

Cognitively demanding instruction (Smith and Stein 1998) is based on the use of mathematical tasks that

allow students to think critically and deeply about mathematical topics, such as that used by Ramos. The mathematics teacher must not only choose cognitively demanding tasks (CDTs) but also retain a high level of cognitive demand throughout the class. A teacher can choose a high-level task but then teach students how to do it procedurally, thus reducing cognitive demand.

Assuming that a teacher has chosen a high-level task, how can he or she sustain cognitive demand for all students? One strategy is to embrace students' use of diverse strategies. Then this diversity is used to plan cognitively demanding instruction that allows all learners to build from their own thinking and access the thinking of their peers to bridge new concepts. CDTs are challenging because of their open nature. However, Ramos's class turned that argument completely on its head. Because these tasks are open, they allow access to students who struggle in mathematics. Being open also implies having more than one entry point, which makes them accessible to students who often struggle to follow one particular procedure.

According to UDL, for tasks to be accessible to a wide range of learners, mathematics classrooms must allow for three aspects:

- Multiple means of representation
- Multiple means of engagement
- Multiple means of strategic action

We will use these three aspects of UDL to analyze how Ramos created a classroom in which students with and without disabilities were able to engage in a cognitively demanding task.

MULTIPLE MEANS OF REPRESENTATION

Students may need different representations of a mathematical task to help them engage. Some students read the problem in **figure 1** and focused on the visual aid; others paid close attention as Ramos read it aloud because the auditory presentation made more sense to them. Ramos read the problem herself, rather than asking a student to read it, thus providing a clear representation for those students who preferred to hear the problem read aloud. This task included a grid, which Erika used immediately. Other students, such as Manuel, did not use the grid but instead relied on what he knew about percentages.

Ramos allowed students to create their own representations, tacitly sending a message that mathematics can be owned by students (Imm, Stylianou, and Chae 2008). As they worked, Ramos looked and listened carefully to their ways of representing the problem. Subsequently, when students were invited to share their thinking, strategies, and solutions, Ramos represented student work both in words and in mathematical notation. She repeated the verbal description of each strategy twice, summarizing after each strategy the steps that the student took.

By revoicing their strategy, she increased the opportunities for other students to engage in that strategy (O'Connor and Michaels 1996). Ramos visually represented the students' approaches herself, rather than having students write them, so that she could create a clear and concise mathematical representation. The board, which became filled with strategies, gave students options to choose from. Later, while solving a problem that incorporated a circle graph and percentages, students referred to these strategies. During interviews, students in this class spoke about how helpful Ramos's representations were for their learning.

Ramos visually represented the students' approaches herself, rather than having students write them, so that she could create a clear and concise mathematical representation.

MULTIPLE MEANS OF ENGAGEMENT

Students were expected to solve this grid problem and participate in mathematical discussion. Both kinds of engagement can be challenging for students. In this routine, students first worked independently, then discussed their work within a pair setting and then within the community. This routine was purposefully designed to allow multiple means of engagement. One student who rarely contributed to whole-group discussion in her math class did her most productive mathematical thinking during pair work. Other students found it stimulating to participate in whole-group discussions.

Students need to feel safe to truly engage in learning. Ramos had created a community in which taking risks was encouraged. Most students shared their work in both small groups and within the class. Ramos validated a variety of strategies, which encouraged a wide range of student participants. She kept this routine consistent

A consistent routine is a critical element in supporting all learners to engage in mathematical thinking at high levels.

throughout the year, supporting students to engage in the ways that were familiar for them yet also encouraging them to engage in practices that were new. A consistent routine is a critical element in supporting all learners to engage in mathematical thinking at high levels (Foote and Lambert 2011).

Ramos reported that she had difficulty at the beginning of the school year with some students who were slow to begin independent problem solving. She created an environment in which sharing a strategy and participating in mathematical discussion were highly valued by both students and teachers. This created motivation for all students to engage deeply to solve each problem. Although the routine began by allowing students time to solve the problem independently, she also gave them time to work with their partners so that those who felt stuck could collaborate with a peer.

MULTIPLE MEANS OF STRATEGIC ACTION

Students were encouraged to use their own way to solve the grid problem, thus allowing for multiple means of strategic action. Ramos assumed that students would have multiple strategies for finding percentages of a number. Students who shared their strategies were not only expressing their ideas but also developing them as they spoke. Student "think alouds" are critical in developing deep understanding of one's own strategies (Siegler and Lin 2010). Ramos used "wait time" effectively, allowing time for students who were slower to express their ideas in words and to formulate their thoughts. This practice is especially critical for students who may need more time to explain their thinking (Foote and Lambert 2011). Her patience communicated to students that speed was not an essential part of doing mathematics.

She also supported action, expression, and engagement by investing significant time to develop students' own questioning. In the students' comments and questions, we heard echoes of the prompts that she had given her students. At the beginning of the school year, Ramos led a discussion on the similarities and differences between various strategies; later, students took on this practice. The use of prompts to support discourse routines was critical in developing metacognitive, strategic skills in students.

One student, Ivana, told us in interviews that she greatly enjoyed the challenge of finding the connections between multiple strategies. Her comment during this lesson, a critique of Erika's strategy, was an example of how heterogeneous classes contribute to student learning. Ivana engaged in comparisons across strategies, which both developed her own thinking and provided a valuable model for other students. Finally, Ramos structured her class so that students had opportunities to not only discuss strategies but also practice their strategies so that they could become fluent with them.

This practice of supporting students to develop their own strategies is echoed in research on the complex development of rational number concepts. Both Erika and Solomon used a strategy of halving and halving again, which can be a bridge between additive and multiplicative thinking (Fosnot and Dolk 2002; Lamon 2007). This bridge can be critical in helping students develop into multiplicative thinkers who can work flexibly with rational numbers.

Strategies that were structurally similar may have appeared very dissimilar to students; Solomon saw his strategy of dividing by 4 as very different from Erika's strategy of finding 50 percent, then 25 percent. When Manuel built a percentage in parts, his complex strategy eventually led to finding 1 percent and multiplying by 36, a link to the commonly taught algorithm. Students will typically begin, as Manuel did, with much larger chunks than 1 percent. Requiring a single strategy erases the unique development along the "landscape of learning" of rational numbers (Fosnot and Dolk 2002). In this case, students who had traditionally struggled in math, even ones who had been identified as having disabilities in mathematics, had brought various strategies, almost all successful, to the classroom. Picking one strategy to teach over others would mean asking students to erase successful past learning.

ONE WAY OF MANY

The demands of the Common Core State Standards in Mathematics can be met through consistent student engagement in CDTs. To allow all learners to access CDTs, we as teachers need to develop the confidence to plan for the wide range of learners in our classrooms. Ramos accomplished this access through cognitively demanding tasks placed within routines that included options for representation, engagement, and strategic action. This was not the *only* way, by any means, to create a mathematics

NCTM 2013 Regional Conferences & Expositions

BALTIMORE, MARYLAND | OCTOBER 16–18

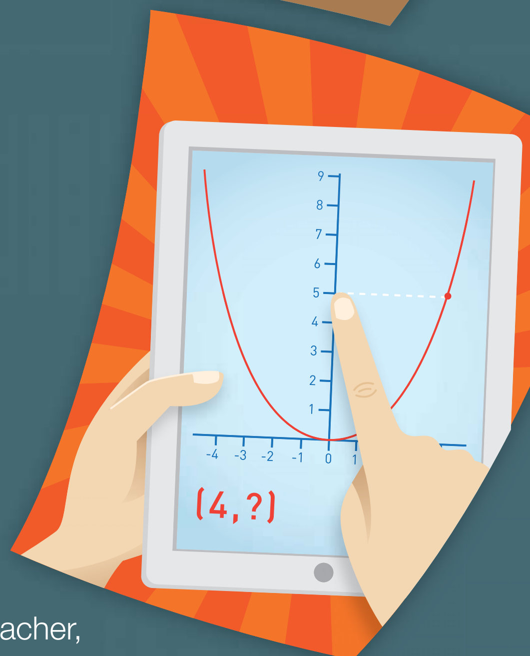
LAS VEGAS, NEVADA | OCTOBER 23–25

LOUISVILLE, KENTUCKY | NOVEMBER 6–8

Help Your Students Succeed in a Competitive World

In a global society with rapidly changing technology your students need the right tools to succeed. So take the next step to help them grow—focus on the latest topics for math education at an NCTM Regional Conference. By attending you and your colleagues will:

- Learn more about the latest **technology**
- Discover new and effective **intervention** methods
- Refine your **assessment** techniques
- And more!



Whether you're a classroom teacher, coach, administrator, preservice teacher, or math specialist, this conference has something for you.

NCTM® NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

(800) 235-7566 | WWW.NCTM.ORG

Visit **www.nctm.org/meetings** for up-to-date information.

Picking one strategy to teach over others would mean asking students to erase successful past learning.

classroom that provided access for a wide variety of learners; it was also not the only way to use cognitively demanding tasks with a wide range of learners. Our objective was to describe one routine to develop strategic thinking through engagement in a cognitively demanding task and demonstrate how the routine allowed a wide variety of learners to gain access to the mathematics.

UDL is not an end product. Rather, it is an open-ended process through which educators rethink who belongs in our mathematics classrooms. Our goal is to create dialogue in the mathematics education community around the design of classrooms that allow all students to participate in rigorous mathematical thinking. Monica, the student who bravely asked why her strategy did not work, told us in an interview that Ramos makes sure that her students "find a way that is our way."

BIBLIOGRAPHY

Center for Applied Special Technology (CAST). 2011. *Universal Design for Learning Guidelines Version 2.0.* Wakefield, MA: CAST.

Foote, Mary Q., and Rachel Lambert. 2011. "I Have a Solution to Share: Learning through Equitable Participation in a Mathematics Classroom." *Canadian Journal of Science, Mathematics and Technology Education* 11 (3): 247–60.

Fosnot, Catherine T., and Maarten Dolk. 2002. *Young Mathematicians at Work: Constructing Fractions, Decimals, and Percents*. Portsmouth, NH: Heinemann.

Imm, Kara Louise, Despina A. Stylianou, and Nabin Chae. 2008. "Student Representations at the Center: Promoting Classroom Equity." *Mathematics Teaching in the Middle School* 13 (April): 458–63.

Lamon, Susan J. 2007. "Rational Numbers and Proportional Reasoning: Toward a Theoretical Framework for Research." In *Second Handbook of Research on Mathematics Teaching and Learning,* 2nd ed., edited by Frank K. Lester Jr., pp. 629–68. Charlotte, NC: Information Age Publishing.

Meyer, A., and David H Rose. 2005. "The Future Is in the Margins: The Role of Technology and Disability in Educational Reform." In *The Universally Designed Classroom: Accessible Curriculum and Digital Technologies*, edited by David H. Rose, pp. 13–35. Cambridge, MA: Harvard University Press.

O'Connor, Mary Catherine, and Sarah Michaels. 1996. "Shifting Participant Frameworks: Orchestrating Thinking Practices in Group Discussion." In *Discourse, Learning, and Schooling,* edited by Deborah Hicks, pp. 63–103. Cambridge: Cambridge University Press.

Siegler, Robert S., and Xiaodong Lin. 2010. "Self-explanations Promote Children's Learning." In *Metacognition, Strategy Use, and Instruction*, edited by Harriet Salatas Waters and Wolfgang Schneider, pp. 85–112. New York: The Guilford Press.

Silver, Edward A., and Mary Kay Stein. 1996. "The QUASAR Project: The Revolution of the Possible in Mathematics Instructional Reform in Urban Middle Schools." *Urban Education* 30 (4): 476–521.

Smith, Margaret, and Mary Kay Stein. 1998. "Selecting and Creating Mathematical Tasks: From Research to Practice." *Mathematics Teaching in the Middle School* 3 (February): 344–50.

Stein, Mary Kay, Barbara W. Grover, and Marjorie Henningsen. 1996. "Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms." *American Educational Research Journal* 33 (2): 455–88.

Stein, Mary Kay, Margaret S. Smith, Marjorie Henningsen, and Edward A. Silver. 2009. *Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development.* New York: Teacher's College Press.

Weiss, Iris R., Joan D. Pasley, P. Sean Smith, Eric R. Banilower, and Daniel J. Heck. 2003. *Looking Inside the Classroom: A Study of K–12 Mathematics and Science Education in the United States*. Chapel Hill, NC: Horizon Research.

The research reported was supported in part by the National Science Foundation (NSF) under grant no. REC-0447542. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the NSF.

Rachel Lambert, rpetunial@gmail.com, who has had over fifteen years of experience as a teacher, is currently a doctoral student in Urban Education at the Graduate Center of the City University of New York, specializing in disability studies and mathematics. **Despina A. Stylianou**, dstylianou@ccny.cuny.edu, a professor of mathematics education at The City College of New York, is interested in the development of students' mathematical practices across the grades, including proof and representation.

Get Published!
Be a Journal Referee.
Avoid Common Writing Pitfalls!

Find out how at the new BuzzHub Stage
located in the exhibit hall.

NCTM's 2013 Annual Meeting and Exposition
April 17–20 • Denver, Colorado

The journal editors from *Teaching Children Mathematics*, *Mathematics Teaching in the Middle School*, and *Mathematics Teacher* will be giving a series of mini-sessions to help you write or referee for one of NCTM's school journals. Inside of 15 minutes, you'll discover how to submit your ideas for publication, volunteer as a referee, or polish an existing manuscript. The editors will explain the peer-review process, answer your questions, point you in the right direction, and allay any fears you may have about getting started. All for a price that can't be beat—free!

Here's what's going on:

Get Published
Discover how simple it is to turn your ideas into articles.
Presented by Sara-Lynn Gopalkrishna, MTMS editor

Thursday, April 18:
10:40–10:55 a.m. and
1:10–1:25 p.m.

Friday, April 19:
10:30–10:45 a.m. and
1:50–2:05 p.m.

Be a Journal Referee
Find out how critiquing manuscripts can help your career.
Presented by Albert Goetz, MT editor

Thursday, April 18:
11:05–11:20 a.m. and
1:35–1:50 p.m.

Friday, April 19:
10:55–11:10 a.m. and
2:15–2:30 p.m.

Avoid Writing Pitfalls
Learn hints on steering clear of those pesky manuscript potholes.
Presented by Beth Skipper, TCM editor

Thursday, April 18:
11:30–11:45 a.m. and
2:00–2:15 p.m.

Friday, April 19:
11:20–11:35 a.m. and
2:40–2:55 p.m.

MATHEMATICS teacher
mathematics teaching in the MIDDLE SCHOOL
teaching children mathematics